Math 561 Worksheet 4

Group members:

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Goal of this worksheet is to understand what the questions are saying for Homework 3, but on a different example. See pages 5-9 of the textbook for the outline, while learning how to best answer these questions yourself.

1. Likelihood inference.

- Write down the likelihood function $L(\theta|D) = \prod p_\theta(j)^{u_j}$.
- Write down the log-likelihood function $l(\theta|D) = \log L(\theta|D)$.
- The maximum likelihood estimate $\hat{\theta}$ is the maximizer of the log-likelihood function:

$$\hat{\theta} = \text{argmax}_{\theta\in\Theta} l(\theta|D).$$

  Unlike the MLE on hw3, this one is easier to derive. Use the formulation above and calculus (yes, take partial derivatives by hand) to derive the MLEs $\hat{\pi}_{ij}, \hat{\alpha}_{ij}, \hat{\beta}_{jk}$ written in equation 1.1.4 in the book.
  - For the observed data $u = (4, 0, 3, 0, 1, 1, 3, 1)$, compute directly the value of the MLE $\hat{p}$ that is written on page 7 of the book.
  - Note this was the same vector that appeared on Homework 1.

2. Evidence for or against the model fit for the two-step Markov chain.

- What is the probability of the observed data $u = (4, 0, 3, 0, 1, 1, 3, 1)$?
- Propose a definition of the p-value for this data set, for the problem of testing the goodness of fit of the Markov chain model.
- What problem do you have to solve explicitly in order to compute this p-value?
  - Explain how this is similar to and different from the setup of problem 2 on homework 3.